A deterrent effect of the electorate’s disappointment on candidates’ over-promise*

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Abstract

We develop a model of campaign promise competition between electoral candidates. Each candidate has a different ability, which determines his achievement if he wins, and which is his private information. Each sends a message about his own ability to the electorate simultaneously, and a generous promise increases a candidate’s winning probability. However, once he wins, an over-promise elicits the electorate’s disappointment and injures his reputation. We find that a higher disappointment tendency is sometimes profitable for the electorate, even taking the disappointment damage into account. This is because it will deter candidates’ over-promise and bring about a more efficient winner selection.

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1 Introduction

An over-promise during an election campaign is a familiar story; even President Barack Obama could not avoid it. While his generous promises (e.g., on health care reform) may have helped him win the 2008 U.S. presidential election, a negative side effect exists. As now widely reported in the media, his over-promise injured his reputation, and this is one of the reasons why his re-election in 2012 was close. That is, while an over-promise in an election increases a candidate’s current winning probability, it will injure his reputation if he wins.

In order to study this intertemporal trade-off relationship, we will develop a model of campaign promise competition by the candidates of an election. We consider that the reputation of the winner is formed by his achievement as well as injured by the electorate’s disappointment. The first goal of this paper is to examine how the electorate’s disappointment affects the candidates’ campaign behavior.

The second is to investigate the welfare consequences of disappointment. While disappointment is itself damaging, the expectation of its existence deters the candidates from over-promising and brings about a more efficient winner selection. Then, it may be profitable for the electorate to have a higher disappointment tendency.

Specifically, we study the following game played by two risk-neutral candidates and the electorate. There is a predetermined policy goal, and the winner of the election will attempt to achieve it.

Each candidate has a different ability, which has a single dimension and is the determinant of his achievement if he wins. Abilities are independently and identically distributed. The realized ability is his private information.

Each sends a message about his ability to the electorate simultaneously; however, this is not necessarily his true ability. The message space is exactly the same as the type space, and therefore the message has caps and limits. We restrict our attention to the class of equilibria in which the electorate chooses the candidate sending the higher message as the winner.
The objective of each candidate is to maximize the expected value of his ex-post reputation. The winner’s reputation is determined by his ability and his message. A high-ability candidate has the potential to have a high reputation. However, if the winner’s message is strictly larger than his true ability, then the gap brings about the electorate’s disappointment, and hence his reputation decreases. The loser obtains no reputation at all.

Our main results are as follows. Firstly, we characterize the equilibrium strategies of the game and provide a sufficient condition under which an over-promise arises. Secondly, we show that a higher tendency of disappointment is sometimes profitable for the electorate because it will serve as a deterrent to over-promise and bring about a more efficient winner selection (in return for the realized damage from disappointment). This result may justify the existence of a disappointment tendency of human beings.

The remainder of the paper is organized as follows: In the next subsection, we describe the related literature. We present the model in Section 2. In Section 3, we characterize the symmetric equilibrium strategy of the game and study the relationship between the electorate’s tendency of disappointment and the expected payoff of the electorate. In Section 4, we conclude the paper.

1.1 Related Literature

In the literature of political competition, one of the most famous models is the Hotelling-Downs spatial competition (Hotelling 1929, Downs 1957). In the model, two parties compete with each other. Each party is non-ideological, and chooses his binding policy platform to maximize his number (or share) of votes obtained. Each voter has different policy preferences, and votes for the party having the most favorable policy. That is, competition is horizontal.¹

Harrington (1993) is the first paper that studies the impact of reelection pressures on the fulfillment of campaign promises. He considers a horizontal compe-

¹This line of research is still ongoing. For example, see Haan and Volkerink (2001) and Matsushima (2007).
tition model with asymmetric information and shows the condition for campaign promises to be informative.\textsuperscript{2}

Another strand of literature on political competition deals with vertical competition. Each party or candidate of an election has a different ability to address a(n exogenous) policy goal. The present paper follows this approach.

Haan et al. (2010) is the paper most closely related to ours. They compare the efficiency result of an appointment by election with that of a random appointment in a model of vertical competition. If the appointment is by election, the electorate can choose the highest-ability candidate, and each candidate chooses an over-ambitious policy platform. On the other hand, under random appointment, while the best candidate is not certain to be chosen, the chosen candidate does not have to set an over-ambitious policy platform. Their main result is that it may be profitable for the electorate to choose a politician randomly.

The present paper studies a similar tradeoff relationship. However, we do not compare an election with random appointment, and our concern is the role of the electorate’s disappointment tendency in an election. In this respect, our paper differs from that of Haan et al. (2010).

This paper is also related to the literature of reference dependent preferences. In the realm of economics, it is usually assumed that a consumer’s utility is a function of his consumption bundle. On the other hand, there is a large body of literature on behavioral economics that deals not only with such “material” payoffs but also with “psychological” ones.

One of the most active topics of research is reference-dependent preferences. It is assumed that the reference point also affects one’s utility.

There are some well known preferences, including the loss aversion preference of Kahneman and Tversky (1979) and the disappointment aversion preference of Gul (1991). In Kahneman and Tversky (1979), the reference point is assumed to be

\textsuperscript{2}Aragonès, Palfrey, and Postlewaite (2007) is another paper that studies the role of reelection pressure on the fulfillment of campaign promises. They studied an infinitely repeated model of horizontal campaign promise competition and characterized the maximal credible promises in the equilibrium.
exogenously given. On the other hand, in Gul (1991), it is the certainty equivalence of lotteries, and therefore is endogenously determined.

Gul (1991) and the present paper share the same terminology, disappointment. However, our definition of disappointment is somewhat different from that of Gul (1991); we assume that the winner’s message becomes the reference point for the electorate.

2 The model

We consider the following campaign promise game. There are three players: candidates 1 and 2 and the electorate (a group of voters).\(^3\)

Each candidate has a different ability. The ability of candidate \(i\) is denoted by \(\theta_i\). This is the rate of attaining a predetermined policy goal when he is elected. Abilities are drawn independently from an interval \([0,1]\) according to the distribution \(F\), which has an everywhere strictly positive density function \(f\) (i.e., \(\lim_{\theta \to 0} F'(\theta) > 0\)). We assume that \(F\) is common knowledge among the candidates and the electorate, and that each candidate’s realized ability is his private information.

Each candidate chooses and sends a message \((m_i \in [0,1])\) about his own ability simultaneously. After observing the messages, the electorate chooses the winner of the election.

The winner is assigned the predetermined task. As described above, his level of achievement of the policy goal is equivalent to his true ability.

The electorate’s payoff is dependent on the achievement level (i.e., the winner’s ability). In addition, the difference between the winner’s message and the realized outcome also affects the payoff if the former exceeds the latter. Specifically, we assume that the electorate’s payoff is

\[
\pi = \theta_i - \alpha \max\{m_i - \theta_i, 0\}
\]  

\(^3\)We assume that all members of the electorate share a mutual interest, so we regard them as a single entity.
when candidate $i$ is the winner. The value of $\alpha (\geq 0)$ represents the strength of the disappointment tendency. We assume that the value is exogenously given and common knowledge. If $\alpha$ is zero, the electorate is only interested in the tangible part, $\pi = \theta_i$.

We regard the electorate’s payoff as the winner’s reputation. We assume that the winner only cares about his reputation. This assumption can also be interpreted as meaning that he cares about the probability of re-election, honor, and other traditional characteristics of reputation. The loser obtains no reputation at all. In summary, the candidate $i$’s payoff is (1) if he wins and zero if he loses.

In this paper, we focus on the sequential equilibrium in which the electorate chooses the candidate sending the highest message as the winner. In the case of a tie, the winner is chosen at random.

Each candidate maximizes his expected reputation. Suppose that rival $j$ uses a strategy $m_j(\cdot)$. Then the expected reputation of candidate $i$ sending $\hat{m}$ can be written as

$$
(\theta_i - \alpha \max\{\hat{m} - \theta_i, 0\}) \left( \Pr(\hat{m} > m_j(\theta_j)) + \frac{\Pr(\hat{m} = m_j(\theta_j))}{2} \right).
$$

There is a tradeoff relationship between the probability of winning and the value of ex-post reputation. A disproportionately higher message yields not only a high probability of winning but also a low ex-post reputation.

3 Analysis

3.1 Competition under no disappointment ($\alpha = 0$)

In this subsection, we first study the case of $\alpha = 0$. In this situation, since the electorate cares only about the winner’s ability, the campaign promise game becomes a cheap talk game.

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4See footnote 6 for other equilibria.

5This situation is similar to the first-price sealed bid auction with bid caps. See, for example, Gravious et al. 2002.
In this case, sending a higher message does not bring about any costs for the candidates. Therefore, each candidate sends the highest message, given that the electorate will choose the candidate sending the higher message as the winner. This means that \( m_1 = m_2 = 1 \) constitutes the equilibrium of the game, and the electorate chooses the winner at random.\(^6\)

Now we can easily obtain the following results. The expected equilibrium payoff of candidate \( i \) having \( q_i \) is \( q_i = 2 \). The ex-ante expected equilibrium payoff of candidate \( i \) (before he knows his ability) is

\[
\frac{1}{2} \int_0^1 \theta f(\theta) d\theta,
\]
and the expected payoff of the electorate is

\[
\int_0^1 \theta f(\theta) d\theta.
\]

### 3.2 Competition under disappointment (\( \alpha > 0 \))

Next we study the case of \( \alpha > 0 \).

#### 3.2.1 An equilibrium over-promise

In Proposition 1, we characterize the message strategy that constitutes the symmetric equilibrium \( (m_1 = m_2 = m) \) under the condition that \( m(\theta|\alpha) \) is larger than \( \theta \) for all \( \theta \in [0, 1] \) (almost everywhere). Without loss of generality, \( m \) is assumed to be right-continuous.

**Proposition 1.** Suppose that the symmetric equilibrium strategy \( m \) satisfies \( m(\theta|\alpha) > \theta \) for \( \theta \in (0, 1) \). Then

\[
m(\theta|\alpha) = \begin{cases} 
\beta(\theta|\alpha) & \text{if } \theta < \bar{\theta}, \\
1 & \text{if } \theta \geq \bar{\theta}.
\end{cases}
\]

Here \( \beta(\theta|\alpha) = \frac{1+\alpha}{\alpha} \left( \theta - \frac{\int_0^\theta F(y) dy}{F(\theta)} \right) \), and \( \bar{\theta} \) satisfies

\[
\frac{\alpha}{1+\alpha} = \bar{\theta} - \frac{2 \int_0^\theta F(y) dy}{1+\alpha}.
\]

\(^6\)An example of other sequential equilibria is the following strategies and belief. The candidates’ strategies are \( m_1(\theta) = m_2(\theta) = 1/2 \) for any \( \theta \), and the electorate’s belief is that if \( m_i = 1/2 \), then \( \theta_i = 1/2 \), and if \( m_i \neq 1/2 \), then \( \theta_i = 0 \).

In this paper, we restrict our attention to the situation where the electorate chooses the candidate sending the highest message as the winner.
Proof. See Appendix.  

Note that \( \beta(q|\alpha) \) is the symmetric equilibrium strategy of the unbounded case (see the proof of Proposition 1). Moreover, the proof tells us the following.

**Proposition 2.** If and only if \( \frac{1+a}{a} \left( \theta - \frac{\int q \cdot F(y)dy}{F(q)} \right) > \theta \) for all \( \theta \in (0,1) \), then \( m(\theta|\alpha) > \theta \) for all \( \theta \in (0,1) \), and \( m \) constitutes the symmetric equilibrium.

**Remark 3.1.** For any \( F \) that satisfies \( \lim_{\theta \to 0} F'(\theta) > 0 \), there exist \( \alpha > 0 \) such that for all \( \alpha < \alpha', \frac{1+a}{a} \left( \theta - \frac{\int q \cdot F(y)dy}{F(q)} \right) > \theta \) for all \( \theta \in (0,1) \).\(^7\)

For the uniformly distributed abilities, the symmetric equilibrium strategy is as follows.

**Example 1.** Suppose that \( F(q) = q \) and \( \alpha = 1 \). Then, \( m(\theta|\alpha) > \theta \) for all \( \theta \in [0,1] \), and the symmetric equilibrium strategy is

\[
m(\theta|\alpha) = \begin{cases} 
\frac{1+a}{a} \theta & \text{if } \theta < \alpha \\
1 & \text{if } \theta \geq \alpha.
\end{cases}
\]

### 3.2.2 The expected payoff of the electorate in the equilibrium

The expected equilibrium payoff of candidate \( i \) having ability \( \theta \) is

\[
u(\theta|\alpha) = \begin{cases} 
(\theta - \alpha(\beta(\theta|\alpha) - \theta))F(\theta) & \text{if } \theta < \bar{\theta} \\
(\theta - \alpha(1 - \theta)) \left( F(\bar{\theta}) + \frac{1-F(\bar{\theta})}{2} \right) & \text{if } \theta \geq \bar{\theta}.
\end{cases}
\]

The ex-ante expected equilibrium payoff of candidate \( i \) (who does not know his own ability) is

\[
\mathbb{E}(u(\theta|\alpha)) \equiv u(\alpha) = \int_{\theta}^{\bar{\theta}} (\theta - \alpha(\beta(\theta|\alpha) - \theta))F(\theta)f(\theta)d\theta \\
+ \int_{\theta}^{1} (\theta - \alpha(1 - \theta)) \left( F(\bar{\theta}) + \frac{1-F(\bar{\theta})}{2} \right) f(\theta)d\theta
\]

\[
= \int_{0}^{\theta} (1 + \alpha) \int_{0}^{\theta} F(y)dyf(\theta)d\theta \\
+ \int_{\theta}^{1} (\theta - \alpha(1 - \theta)) \frac{1+F(\bar{\theta})}{2} f(\theta)d\theta,
\]

\(^7\)Recall that we have assumed that the distribution function \( F \) has an everywhere strictly positive density function \( f \), and hence \( \lim_{\theta \to 0} F'(\theta) > 0 \).
where $\mathbb{E}$ is the expectation operator. The expected equilibrium payoff of the electorate is $\mathbb{E}(\pi) = 2u(\alpha)$.

For the uniformly distributed abilities:

**Example 2.** If $F(\theta) = \theta$ (and $\alpha \leq 1$),

$$u(\theta|\alpha) = \left\{ \begin{array}{ll}
\frac{(1+\alpha)^2\theta}{2} & \text{if } \theta < \alpha \\
\frac{(1+\alpha)^2}{2} - \frac{(1+\alpha)\alpha}{2} & \text{if } \theta \geq \alpha,
\end{array} \right.$$ 

and

$$u(\alpha) = \frac{1}{4} + \frac{\alpha^3}{6} - \frac{\alpha^4}{12},$$

and

$$\mathbb{E}(\pi) = \frac{1}{2} + \frac{\alpha^3}{3} - \frac{\alpha^4}{6}.$$
The derivative of $u(\alpha)$ is as follows.

$$u'(\alpha) = -\left( \int_0^\theta (\beta(\theta|\alpha) - \theta) F(\theta) f(\theta) d\theta + \int_\theta^1 (1 - \theta) \frac{1 + F(\theta)}{2} f(\theta) d\theta \right)$$

$$+ \int_0^\theta \alpha \left( -\frac{\partial \beta(\theta|\alpha)}{\partial \alpha} \right) F(\theta) f(\theta) d\theta + \bar{\theta}' f(\bar{\theta}) F(\bar{\theta}) (1 - \beta(\bar{\theta}|\alpha))$$

$$+ \alpha \bar{\theta}' f(\bar{\theta}) \frac{1 - F(\bar{\theta})}{2} \left( \int_\bar{\theta}^1 \theta \frac{f(\theta)}{1 - F(\theta)} d\theta - \bar{\theta} \right)$$

$$+ \bar{\theta}' f(\bar{\theta}) \frac{1 - F(\bar{\theta})}{2} \left( \int_\bar{\theta}^1 \theta \frac{f(\theta)}{1 - F(\theta)} d\theta - \bar{\theta} \right)$$

- (2) corresponds to the direct effect of an increase in the unit disappointment cost $\alpha$. That is, given a gap between a message and actual ability, a unit cost increase simply increases the total cost caused by the gap.

- A unit cost increase also has indirect effects. That is, a unit cost increase also causes a change in the equilibrium message. The statement change also has direct and indirect effects. (3) corresponds to the direct effect of the statement change. That is, given a winner, the winner states a lower message (and hence causes a smaller gap).

- The statement change also causes a change in who wins, specifically at the maximum message, by causing a dropout with ability $\bar{\theta}$ from the maximum message. By the dropout, a loser of the tie-break becomes the winner, who has a higher ability. Then, regarding the disappointment cost, the gap from the maximum message becomes less, corresponding to (4).

- Moreover, the ability of the winner becomes higher, corresponding to (5).

The first effect has negative impact on the expected payoff of the electorate, while the others have positive effects. In the uniformly distributed case (Example 2), the positive effects dominate the negative ones. This is the reason why $E(\pi)$ is increasing in $\alpha$ in this case.

Figure 1 depicts the marginal effects of an increase in $\alpha$ in the uniformly distributed case. The black line corresponds to the overall effect $\frac{\partial u(\alpha)}{\partial \alpha}$, the red to (2), the green to (3), the light blue to (4), and the blue to (5).
Figure 1: The effects of a change in $\alpha$ on the expected payoff of the electorate.

It is natural to imagine that if there is a case in which the negative effects dominate the positive effects, we can obtain the opposite result. Therefore, we will try to construct an example of this.

3.2.4 $\alpha = 0$ can be optimal

In this subsection, we will present an example that shows that $\alpha = 0$ can be optimal (for the electorate) when the disappointment tendency cannot be so high that no candidate will send the highest message, 1 (unless his ability is exactly 1). That is, $\alpha = 0$ can be (uniquely) optimal among the set $\{ \alpha | m(\theta | \alpha) = 1 \text{ for some } \theta < 1 \}$.

We will study the situation of

$$F(\theta) = \begin{cases} \frac{\theta}{1 - \varepsilon}, & \text{if } \theta \leq \frac{1 - \varepsilon}{2}, \\ \frac{\theta}{2} + (\theta - \frac{1 - \varepsilon}{2}) \frac{1 - \varepsilon}{1 - \varepsilon}, & \text{if } \theta \in \left( \frac{1 - \varepsilon}{2}, \frac{1 + \varepsilon}{2} \right], \\ \frac{\theta}{2} + 1 - \varepsilon + (\theta - \frac{1 + \varepsilon}{2}) \frac{\varepsilon}{1 - \varepsilon}, & \text{if } \theta > \frac{1 + \varepsilon}{2}, \end{cases}$$

and we assume that $\varepsilon \geq 1/2$. In this case, irrespective of $\varepsilon \geq 1/2$, the set is $[0, 1)$.

Note that if $\varepsilon = 1/2$, this is the uniform distribution function.

We can calculate the symmetric equilibrium strategy according to Proposition 1. For example, the strategy under $\alpha = 0.9$ and $\varepsilon = 0.3$ is depicted in Figure 2.

\[^8\text{Note that if the value of } \alpha \text{ is sufficiently large, } m(\theta | \alpha) \text{ becomes } \theta; \text{ the electorate has a much higher disappointment tendency, while the candidates tell the truth. Then, the electorate can achieve both efficient winner selection and damage avoidance.}\]
The expected equilibrium payoff of the electorate $\mathbb{E}(\pi)$ under $\varepsilon = 0.3$ is depicted in Figure 3. The horizontal axis is $\alpha$ and the vertical axis is $\mathbb{E}(\pi)$. This example tells us that $\alpha = 0$ is optimal if the value of $\alpha$ can be taken in the interval of $(0, 1)$.

![Figure 2: The symmetric equilibrium strategy: $\alpha = 0.9$ and $\varepsilon = 0.3$.](image)

![Figure 3: The expected equilibrium payoff of the electorate: $\varepsilon = 0.3$.](image)

Figure 4 depicts the marginal effects of an increase in $\alpha$ in this example. The black line corresponds to the overall effect $\frac{\partial u(\alpha)}{\partial \alpha}$, the red to $(2)$, the green to $(3)$, the light blue to $(4)$, and the blue to $(5)$. 12
Figure 4: The effects of a change in $\alpha$ on the expected payoff of the electorate.

4 Conclusion

This paper studied a model of a campaign promise game played by two candidates and the electorate. We assume that each candidate sends a message about his ability to the electorate simultaneously, and that a generous promise increases his winning probability. However, once he wins, a disproportionately higher message elicits the electorate’s disappointment and injures his reputation.

This reputation concern effect works well if the electorate’s disappointment tendency is higher. Therefore, a higher disappointment tendency is sometimes profitable for the electorate, even taking the disappointment damage into account. This is because it will deter the candidates from over-promising and bring about a more efficient winner selection.

In this paper, we assumed for simplicity that the number of candidates is two. In the real world, however, this number is more than two in many situations. We can extend our model to include many candidates to, for example, study the relationship between the number of candidates and the expected payoff of the electorate. We plan to study this topic in another paper.
Appendix

Proof of Proposition 1. Let us consider the message game without bid caps. Suppose that for all $\theta \in [0,1]$, the equilibrium message $m(\theta) > \theta$ (almost everywhere). In this case, by using the standard method in the auction theory, we can obtain the symmetric equilibrium strategy as follows:

$$
\beta(\theta) = \frac{1 + \alpha}{\alpha} \left( \theta - \frac{\int_0^\theta F(y)dy}{F(\theta)} \right).
$$

Since the type space is $[0,1]$ and the message space is also $[0,1]$, we should care about whether $\beta(1) \geq 1$ or not. The inequality $\beta(1) \geq 1$ is equivalent to

$$
\frac{1 + \alpha}{\alpha} \left( 1 - \int_0^1 F(y)dy \right) \geq 1.
$$

Thus, we obtain the threshold value of $\alpha$:

$$
\hat{\alpha} \equiv \frac{1 - \int_0^1 F(y)dy}{\int_0^1 F(y)dy}.
$$

If $\alpha < \hat{\alpha}$, then “bunching” occurs.

In the symmetric equilibrium strategy $m(\cdot)$, there is a threshold value of $\theta$, $\tilde{\theta}$, such that all types in the interval of $[\tilde{\theta}, 1]$ choose $m = 1$ if $\alpha \leq \hat{\alpha}$. Therefore, we will find the value of $\tilde{\theta}$.

Suppose that rival $j$ uses the following strategy:

$$
m_j(\theta) = \begin{cases} 
\beta(\theta) & \text{if } \theta \in [0, \tilde{\theta}] \\
1 & \text{if } \theta \in [\tilde{\theta}, 1].
\end{cases}
$$

We can find the threshold value, $\tilde{\theta}$, such that candidate $i$ having ability $\tilde{\theta}$ is indifferent about choosing between $m = \beta(\tilde{\theta})$ and $m = 1$.

If the message is $\beta(\tilde{\theta})$, his expected payoff is

$$
F(\tilde{\theta}) \left( \tilde{\theta} - \alpha \left( \beta(\tilde{\theta}) - \tilde{\theta} \right) \right) = (1 + \alpha) \int_0^{\tilde{\theta}} F(y)dy. \quad (6)
$$

On the other hand, when candidate $i$ with ability $\tilde{\theta}$ sends the message of 1, his probability of winning is

$$
\left( F(\tilde{\theta}) + \frac{1 - F(\tilde{\theta})}{2} \right) \left( \tilde{\theta} - \alpha (1 - \tilde{\theta}) \right) = \frac{1 + F(\tilde{\theta})}{2} (1 + \alpha) \tilde{\theta} - \alpha. \quad (7)
$$
The condition of \((6) = (7)\) is equivalent to

\[
\frac{\alpha}{1+\alpha} = \bar{\theta} - \frac{\int_{0}^{\hat{\theta}} F(y)dy}{1 - F(\bar{\theta})}.
\]

Thus, we obtain the function that characterizes \(\bar{\theta}\).

**References**


