# Overconfidence in Economic Contests*

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**Abstract**

This paper studies an economic contest with two participants, who are overconfident in their own relative abilities. We examine two different sources of overconfidence, overestimation of one's own ability and underestimation of the rival's ability, and compare the behavioral consequences of each situation with the correctly estimated case. The main result is that the former always induces the participants' aggressive behavior, whereas the latter sometimes brings about less aggressive behavior of one or both participants.

**JEL classification numbers:** D44, D82.
**Keywords:** Overconfidence, economic contest, relative ability, all-pay auction, distribution of abilities.

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# 1 Introduction

Overconfidence is one of the famous stylized facts about human behavior. Several studies in psychology and experimental economics have shown that humans are overconfident in their own (relative) abilities. For example, Svenson (1981) reported that almost all drivers in Texas believe that their own driving skills are above average, and he concluded that a considerable number of subjects were overconfident in their own relative driving skills.

In addition, these studies usually indicate that overconfidence induces aggressive behavior. For example, Camerer and Lovallo (1999) used an experimental approach and showed that overconfidence commonly induces excess entry into markets and business failure.[1]

The aim of this paper is to study the behavioral consequences of participants' overconfidence in economic contests with two participants. Well known applications of economic contests are job promotion in internal labor markets, political elections, rent seeking activities, and R&D rivalry. In this paper, we examine the situations in which each participant is overconfident in his/her relative ability. Note that there are some possible situations where a participant is overconfident in his/her relative ability. Overconfidence in relative ability comes from overestimating one's own ability and/or underestimating the rival's ability.

To simplify the analysis, we examine two extreme situations, where each participant has overestimated his/her own ability or underestimated the rival's ability, and compare the behavioral consequences of each situation with the correctly estimated case. By this comparison, we will show that the different sources of overconfidence have different behavioral consequences. More precisely, we will show that in the former case overconfidence always induces aggressive behavior by the participants, whereas in the latter case it sometimes brings about less aggressive behavior by one or both participants.

In this paper, we study the following contest game. The principal hires two risk neutral agents for a specific period and assigns a task to each of them. For exposition, we suppose that the principal is female and the agents are male. Each agent outlays his effort for winning. Agents differ in ability. Higher ability agents can obtain higher returns when they win the contest. We refer to agents' abilities as their types, so each type is equal to the monetary value of their winning the contest. Examples of such situations are the aforementioned promotion contests in firms and political elections. The types are independently and identically distributed.

An agent has prior beliefs about his own type and the distribution of types. In a standard model of economic contest called the "all-pay auction" with incomplete information, the prior (or subjective) beliefs are correct.[2]

[1] See Camerer (1997) for further references.

[2] The difference between the "first price sealed bid auction" and the "all-pay auction" is the following. In the first price sealed bid auction, the winner is the only person who

Moreover, it is assumed that the setting of the game is common knowledge. However, as we already know, overconfidence is frequently observed in the real world. So, in this paper, we will consider a model of economic contests that includes participants' overconfidence.

We consider situations where an agent's beliefs about his own type and the distribution of types may or may not be correct. More precisely, the agent has incorrect information about either his own type or the distribution of types.[3] Here, we assume that the agents do not know that they and their rival are overconfident in their own types ex ante.[4] Each agent's effort is chosen to maximize his expected profit on the basis of prior (or subjective) information. The effort level of each agent is observable by all players at the end when the agents have already chosen their effort levels. For the principal, larger efforts are profitable, so that she welcomes a larger expected effort per agent.

Our results are as follows. First, we derive the symmetric Bayesian Nash equilibrium strategy in the contest game without the agents' overconfidence as the benchmark. Because the contest game considered here is the standard all-pay auction, we can identify the symmetric equilibrium effort strategy and derive the expected effort level per agent by using the standard method used in auction literature. The equilibrium effort levels are strictly increasing with respect to types, so that the probability of winning in the contest is equivalent to the probability that one's own type is no lower than the rival's type.

Second, we show that overestimation of one's own type always increases one's effort and, therefore, it is profitable for the principal. The reason overestimation of one's own type increases his effort is as follows. In the situation of overestimation of one's own type, the agent behaves as if he has a higher type. Since the effort strategy is increasing in types, he chooses an effort that is higher than that in the benchmark case.[5]

Third, we show that underestimation of the type distribution[6] increases

has to pay his own bid. However, in the all-pay auction, all bidders have to pay their own bid. Note that promotion contests in firms and political elections can be regarded as the all-pay auctions because all bidders (participants) have to pay their own bid (effort), irrespective of winning or losing.

[3]Under an auction setting called the all-pay auction under incomplete information, we formulate the situation where a participant underestimates the rival's ability by a dominance relationship between the true distribution of types and the subjective belief about the type distribution (the definition of a dominance relationship is described in Section 3.3). This means that the participant believes that there are more lower ability types than would occur in the true distribution of types.

[4]An agent may find his and/or his rival's overconfidence by the realized value of winning the contest and the realized effort levels ex post.

[5]Note that each player believes that his strategy is an "equilibrium" strategy; the effort strategies chosen by the players do not constitute an equilibrium, however.

[6]We use the terms "underestimation of the type distribution" instead of "underestimation of the rival's ability" here. See footnote 3.

the effort of an agent of low ability, whereas it decreases the effort of an agent of high ability. Therefore, it may or may not be profitable for the principal. Additionally, we show that underestimation increases an agent's expected efforts in some cases, but decreases it in other cases. Thus, the aggregate effect of underestimation is ambiguous. The fundamental reason why underestimation of the type distribution changes each type's effort is that underestimation changes each type's subjective probability of winning.

There are two strands of theoretical works related to the present paper. The first is the analysis of economic contests as a variation of the all-pay auction. Examples are Amann and Leininger (1996), Krishna and Morgan (1997), and Che and Gale (2000).[7] Amann and Leininger (1996) analyzed an asymmetric all-pay auction and Krishna and Morgan (1997) analyzed an all-pay auction with affiliated types. Che and Gale (2000) studied the probability of winning function in economic contests carefully and they studied the all-pay auction as a special case in their contests. Recently, contest design problems from the viewpoint of the contest designer have attracted much attention. Examples include Singh and Wittman (2001) and Ando (2004).[8] These papers and the present paper focus on the fact that changes in each type's probability of winning yield the changes in each type's equilibrium behavior. However, these studies, other than the present paper, assumed that the prior beliefs about one's own type and the type distribution are correct. Hence, the present paper is very different from these studies.

The second strand is the study of overconfidence in the field of behavioral economics.[9] Most studies of overconfidence focused on situations of one person's decision-making. Moreover, these studies usually conclude that overconfidence brings about aggressive behavior by an agent. For example, Dubra (2003) studied a search model with an optimistic individual. In contrast to these studies, the present paper studies an auction setting incorporating overconfidence and we show that overconfidence sometimes yields less aggressive behavior by an agent.

The rest of this paper is as follows. Section 2 describes the environment of our model. Section 3 examines in turn the benchmark case, the behavioral consequences of overestimation of one's own type, and the behavioral consequences of underestimation of the type distribution. Section 4 contains our conclusions.

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[7] Noussair and Silver (2003) is an experimental study of all-pay auctions under incomplete information. We will refer again to Noussair and Silver (2003) in Section 4.

[8] Singh and Wittman (2001) studied the contest in which each agent's output exhibits non-increasing returns to his effort. Additionally, it is assumed that the participant with the highest output need not win. They described the properties of optimal contests and show that, for an open interval of types, an optimal contest uses the rule that the agent with the highest output wins. Ando (2004) studied an economic contest with identical prizes and described the effects of symmetric division of the contest on the agents' behavior.

[9] Camerer (2003) is a useful reference on behavioral game theory. Itoh (2004) is a study toward the behavioral contract theory.

## 2 Model

We consider an economic contest with two risk neutral agents, agents 1 and 2. They compete to win the contest. Each agent $i$ decides his effort $e_i$.[10] Efforts are outlaid simultaneously and independently. The agents' effort levels are observable by the principal and the agents at the end when the agents have already chosen their effort levels.

Each agent has a different type, which represents the monetary value of his win. The true type of agent $i$ is denoted by $\theta_i$ and we assume that the exact value can be perceived only after winning. Moreover, we assume that no one knows the exact value of $\theta_i$ ex ante. We assume that agent $i$ has a prior (or subjective) belief about his type, $a_i$, instead of the knowledge about his true type $\theta_i$, and $a_i$ is his private information. The true type of each agent is drawn independently from an interval $[0, 1]$ according to the distribution function $F$ that has a continuous and everywhere strictly positive density function $f$.[11] However, both agents may not know the shape of the distribution function $F$.

In the standard model of economic contests based on all-pay auctions, each agent's prior belief about his own type is exactly the same as the true type and the shape of the distribution function is assumed to be common knowledge for the principal and both agents. However, in this paper we relax these assumptions. Agent $i$ believes that his type is $a_i$. However, it may not be equal to the true type $\theta_i$ from the viewpoint of us (i.e., the analysts of the model). Moreover, the agent believes that the rival $j$'s prior belief about $j$'s type (i.e., $a_j$) drawn from an interval $[0, 1]$ according to the distribution function $G_i$.[12] However, $G_i$ may not be equal to the true distribution $F$ from our viewpoint. Additionally, the agent $i$ believes that the rival $j$'s prior belief about the type distribution is also $G_i$, and that the above-mentioned belief structures are common knowledge. Here, we assume that each agent does not know the fact that both agents may have incorrect information ex ante. Note that we do not assume that both agents have the same wrong belief each other. Our assumptions mean that an agent believes that his prior belief is the common prior belief among the players. This setting enables us to use the technique of comparative statics.

In this contest, the real payoff of agent $i$ is, $\theta_i - e_i$ if he wins, and $-e_i$ if he does not. However, each agent chooses his effort in order to maximize his expected payoff on the basis of his prior beliefs about his own type $a_i$ and the type distribution $G_i$. Therefore the effort strategies chosen by the

[10]For exposition, we suppose that the principal is female and the agents are male.

[11]We assume that the type space is $[0, 1]$. This restriction is only for analytical convenience. We can preserve all our results in any non-negative type with bounded support cases.

[12]We assume that $G_i$ has a continuous and everywhere strictly positive density function $g_i$.

agents may not constitute an equilibrium, whereas each agent believes that his strategy is an equilibrium strategy. For the principal, larger efforts are profitable, so that she welcomes a larger expected effort per agent.

# 3 Analysis

## 3.1 The benchmark case

In this subsection, we consider the standard all-pay auction as the benchmark case. That is, we consider the situation where $a_i = \theta_i$ and $G_i = F$, $i = 1, 2$, from the viewpoint of the analysts of the model.

In Proposition 1, we show the symmetric equilibrium effort strategy in this contest game. The effort strategy in the benchmark case is denoted by $\beta(\theta, F)$ instead of $\beta(a, G_i \mid a = \theta \text{ and } G_i = F)$ to simplify the notation.

**Proposition 1.** *In a symmetric equilibrium, the effort strategy is*

$$\beta(\theta, F) = \int_0^\theta y f(y) dy, \tag{1}$$

*and the symmetric equilibrium is unique.*

*Proof.* The equilibrium strategy and its uniqueness can be easily derived with the standard method in the auction literature. Hence, the proof is omitted. ∎

In the symmetric equilibrium, since $\beta(\cdot, F)$ is strictly increasing, agent $i$'s probability of winning (hereafter $p(\theta_i, F)$) is equivalent to the probability that his type is no lower than the rival's type; that is, $p(\theta_i, F) = F(\theta_i)$.

Note that, for all $\theta \in [0, 1]$, the gradient of the probability of winning function reflects on the gradient of the equilibrium effort strategy, since $\partial\beta(\theta, F)/\partial\theta = \theta f(\theta) = \theta\partial p(\theta, F)/\partial\theta$. This is an important characteristic of the equilibrium strategy. We will use this later.

In this contest, the expected effort per agent is

$$E(e, F) = \int_0^1 \beta(\theta, F) f(\theta) d\theta. \tag{2}$$

By using integration by parts with expressions (1) and (2), we obtain the following.

**Proposition 2.** *The expected effort per agent is*

$$E(e, F) = \int_0^1 (1 - F(\theta))\theta f(\theta) d\theta. \tag{3}$$

We provide an example.

**Example 3.1.** *If $F(\theta) = \theta$, $\beta(\theta, F) = \theta^2/2$ and $E(e, F) = 1/6$.*

### 3.2 Overestimation of one's own type

In this subsection, we examine the consequences of overestimation of one's own type in economic contests. We assume that agent $i$'s prior knowledge about the type distribution is correct (i.e., $G_i = F$). He believes that his type is $a_i$; however, it is incorrect information from the viewpoint of the analysts of the model. Now, we define the following.

**Definition 1.** *Agent $i$ overestimates his own type if, for given $\theta_i \in [0, 1)$, $a_i > \theta_i$.*

In this situation, agent $i$ derives the effort strategy $\beta$ by $G_i (= F)$. Then, he chooses his action by calculation with $\beta(\cdot, F)$ and $a_i$. From the above facts, we obtain the following.

**Proposition 3.** *Compared with the benchmark case, overestimation of one's own type increases his effort (i.e., $\beta(a_i, F) > \beta(\theta_i, F)$ for all $\theta_i \in [0, 1)$ and for all $a_i > \theta_i$).*

*Proof.* The effort strategy $\beta$ is constructed by $F$ and, therefore, $\beta(a, F) = \int_0^a yf(y)dy$. Since $yf(y)$ is positive for all $y \in (0, 1]$, $\int_0^a yf(y)dy > \int_0^\theta yf(y)dy$ for all $a > \theta$. Thus, we obtain $\beta(a_i, F) > \beta(\theta_i, F)$. ▌

This proposition implies that, in the situations of overestimation of one's own type, an agent who has the true type $\theta_i$ behaves as if he has a higher type, $a_i$. Since the equilibrium strategy is increasing in types, he chooses a higher effort compared with the benchmark case. Thus, overconfidence from overestimation of one's own type is always profitable for the principal.

We provide an example. In this example, an agent with the true type $\theta \in (0, 1)$ has a type overestimation that is constructed by the following simple rule, $a = \sqrt{\theta}$.[13] This formulation permits us to directly calculate the expected effort level.

**Example 3.2.** *Suppose that agent $i$'s prior information is $(a_i, G_i)$ that satisfies $G_i(\theta) = F(\theta) = \theta$ and $a_i = \sqrt{\theta_i}$. In this situation, his effort strategy is $\beta(a_i, F) = \theta_i/2$ and the expected effort level is $E(\beta(a_i, F), F) = 1/4$. If agent $i$ knows his true type, he should follow the effort strategy $\beta(\theta_i, F) = {\theta_i}^2/2$ and the expected effort level is $E(\beta(\theta_i, F), F) = 1/6$.*

The effort strategies in the above example are depicted in Figure 1. The horizontal axis is the true types and the vertical axis is the effort levels.

[13]Note that we know this relationship between one's prior information about type and true type, whereas the players in this game do not know it.

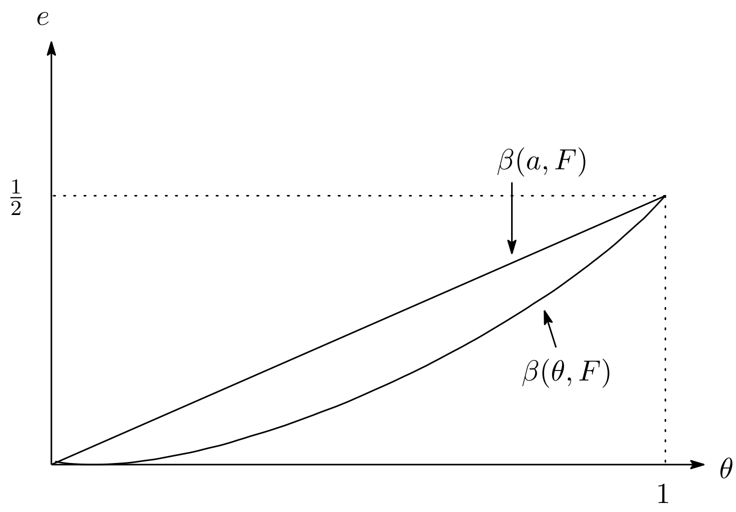


Figure 1:

### 3.3 Underestimation of the type distribution

In this subsection, we examine the consequences of underestimation of the type distribution. We assume that agent $i$'s prior information about his type is correct; that is, $a_i = \theta_i$. However, he has incorrect information about the type distribution (i.e., $G_i \neq F$). To simplify the notation, we remove the subscript $i$ from $G_i$ hereafter.

The next proposition shows agent $i$'s effort strategy under incorrect information about the type distribution.

**Proposition 4.** *If agent $i$ has incorrect information about the type distribution, he chooses his effort according to the following strategy:*

$$\beta(\theta, G) = \int_0^\theta yg(y)dy. \tag{4}$$

*Proof.* The derivation of the strategy is the same as that in Proposition 1. ∎

In this situation, the expected effort is

$$E(e, G) = \int_0^1 \beta(\theta, G)f(\theta)d\theta. \tag{5}$$

Note that the expected effort is derived from the following facts. Each type's effort level is calculated by $\beta(\cdot, G)$ and $\theta_i$. However, the expectation is based on the true distribution of types, $F$.

By using integration by parts with expressions (4) and (5), we obtain the following.

**Proposition 5.** *If agent $i$ has incorrect information about the type distribution, his expected effort level is*

$$E(e,G) = \int_0^1 (1 - F(\theta))\theta g(\theta) d\theta. \tag{6}$$

Now, we turn our attention to the characteristics of incorrect information about the type distribution. We define the following.

**Definition 2.** *For $G \neq F$, agent $i$ underestimates the type distribution if $F$ first order stochastically dominates $G$; that is,*

$$\forall \theta \in [0,1], G(\theta) \geq F(\theta).$$

An intuitive explanation of underestimation is that agent $i$ believes that there are more lower types than would occur in the true distribution of types.

Next, we describe how underestimation changes agents' behavior.

**Proposition 6.** *Compared with the benchmark case, underestimation of the type distribution changes agent $i$'s behavior as follows. For $G \neq F$,*

1. *there exists $\hat{\theta} \in (0,1)$ such that, for all $\theta \in (\hat{\theta}, 1]$, $\beta(\theta, G) < \beta(\theta, F)$, and*

2. *there exists an interval of types $(\theta_a, \theta_b)$ such that for all $\theta \in (\theta_a, \theta_b)$, $\beta(\theta, G) > \beta(\theta, F)$.*

*Proof.* The proof of the former statement is as follows. $\beta(\theta, G) - \beta(\theta, F) = \int_0^\theta yg(y)dy - \int_0^\theta yf(y)dy$. By using integration by parts we obtain $\beta(\theta, G) - \beta(\theta, F) = \theta(G(\theta) - F(\theta)) - \int_0^\theta (G(y) - F(y))dy$. This is strictly negative at $\theta = 1$, since $\theta(G(\theta) - F(\theta))$ is zero and $\int_0^\theta (G(y) - F(y))dy$ is strictly positive at $\theta = 1$ by definition. From the fact that $\beta$ is increasing and continuous, we can conclude that there exists $\hat{\theta} \in (0,1)$ such that, for all $\theta \in (\hat{\theta}, 1]$, $\beta(\theta, G) < \beta(\theta, F)$.

The proof of the latter statement is as follows. We define $\theta_c = \inf\{\theta \mid G(\theta) > F(\theta)\}$. For any $\theta \in [0, \theta_c)$, $\beta(\theta, G) = \beta(\theta, F)$, since for any $\theta \in [0, \theta_c)$, $g(\theta) = f(\theta)$, and $\beta(\theta, G) = \int_0^\theta yg(y)dy$ and $\beta(\theta, F) = \int_0^\theta yf(y)dy$. For sufficiently small $\varepsilon > 0$, $\beta(\theta_c + \varepsilon, G) - \beta(\theta_c + \varepsilon, F) = \int_{\theta_c}^{\theta_c + \varepsilon} y(g(y) - f(y))dy$ and this is strictly positive, since $g(\theta_c + \varepsilon) > f(\theta_c + \varepsilon)$ by definition of $\theta_c$. From the fact that $\beta$ is increasing and continuous, we can conclude that there exists an interval of types $(\theta_a, \theta_b)$ such that for all $\theta \in (\theta_a, \theta_b)$, $\beta(\theta, G) > \beta(\theta, F)$. ∎

The implication of the above proposition is that, if an agent underestimates the type distribution, some types over work and other types under work compared with the benchmark case. Note that there are cases that the sign of $\beta(\theta, G) - \beta(\theta, F)$ changes more than once. However, to simplify the exposition, in the rest of this paper we restrict our attention to situations where $\beta(\theta, G)$ and $\beta(\theta, F)$ are single-crossing in the interval $(0, 1)$ (that is, $\theta_a = 0$ and $\theta_b = \hat{\theta}$).

We can describe a sufficient condition for single-crossing of $\beta(\theta, G)$ and $\beta(\theta, F)$.

**Proposition 7.** *$\beta(\theta, G)$ and $\beta(\theta, F)$ are single-crossing in the interval $(0, 1)$, if there exists $\bar{\theta} \in (0, 1)$, such that $\forall \theta < \bar{\theta}$, $f(\theta) < g(\theta)$, and $\forall \theta > \bar{\theta}$, $f(\theta) > g(\theta)$.*

*Proof.* $\beta(\theta, G) - \beta(\theta, F) = \int_0^\theta y(g(y) - f(y))dy$. This is positive for the type close to zero, since $g(y) > f(y)$ at $y = 0$. Moreover, $g(y)$ and $f(y)$ are crossing only once at a certain $y \in (0, 1)$ and at the left hand side, $g(y) > f(y)$, and at the right hand side, $g(y) < f(y)$. Since $\int_0^1 (g(y) - f(y))dy = 0$ and $y$ is strictly increasing function, we obtain $\int_0^1 y(g(y) - f(y))dy < 0$. This implies $\beta(\theta, G) < \beta(\theta, F)$ at $\theta = 1$.

Next, we show that the magnitude relationship between the gradient of $\beta(\theta, G)$ and that of $\beta(\theta, F)$ changes only once in $(0, 1)$. More precisely, $\exists \acute{\theta} \in (0, 1)$, $\forall \theta < \acute{\theta}, \partial\beta(\theta, G)/\partial\theta > \partial\beta(\theta, F)/\partial\theta$ and $\forall \theta > \acute{\theta}, \partial\beta(\theta, G)/\partial\theta < \partial\beta(\theta, F)/\partial\theta$, since $\partial\beta(\theta, G)/\partial\theta = \theta g(\theta)$ and $\partial\beta(\theta, F)/\partial\theta = \theta f(\theta)$.

From the above and the fact that $\beta$ is increasing and continuous, we can conclude that $\beta(\theta, G)$ and $\beta(\theta, F)$ are single-crossing in the interval $(0, 1)$. ∎

This proposition tells us that, if the density functions $g$ and $f$ are single-crossing, $\beta(\theta, G)$ and $\beta(\theta, F)$ are also single-crossing in the interval $(0, 1)$.

We briefly describe the reason that underestimation of the type distribution changes an agent's effort under the single-crossing situations. When an agent with type $\theta$ underestimates the type distribution, his subjective probability of winning the contest is changed from $F(\theta)$ to $G(\theta)$. In this underestimated situation, since the probability of winning at the left tail (note that the probability of winning for an agent with the lowest ability is always zero) increases faster compared with the benchmark case, underestimation induces more aggressive effort at the left tail. For the higher types, underestimation decreases the gradient of the probability of winning function. This reflects on decreases in the gradient of the effort strategy function for the higher types.

A more intuitive explanation is as follows. Since underestimation of the type distribution implies that the (subjective) probability of winning is higher for every type, an agent with a lower type has a non-negligible probability of winning and therefore he increases his effort. On the other hand, an agent with a higher type (almost) certainly wins and therefore he can slightly decrease his effort.

We provide an example.

**Example 3.3.** *Suppose that agent i's prior information is* $(a_i, G)$ *that satisfies* $a_i = \theta_i$ *and* $G(\theta) = 2\theta - {\theta_i}^2$*, and that* $F(\theta) = \theta$*. In this situation, his effort strategy is* $\beta(\theta, G) = \theta^2 - 2\theta^3/3$*. If he knows the true distribution of types, he should follow the strategy* $\beta(\theta, F) = \theta^2/2$*.*

The above example is depicted in Figure 2. The horizontal axis is the types and the vertical axis is the effort levels.

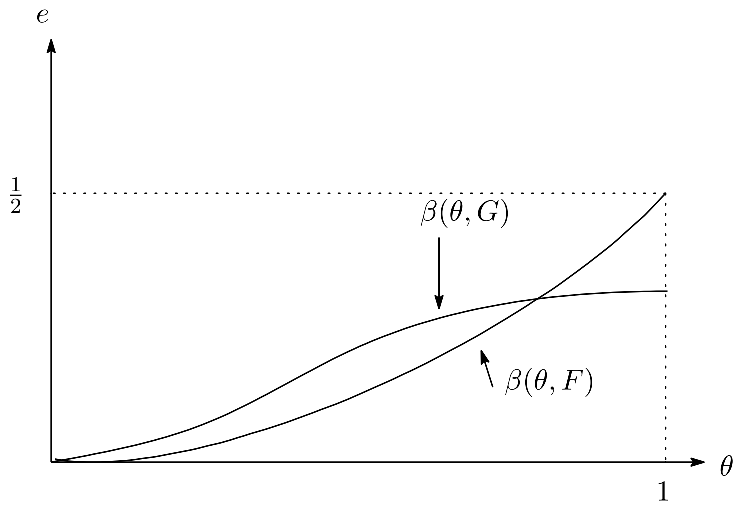


Figure 2:

Finally, we describe the aggregate effect of underestimation of the type distribution on the expected effort level. In the following proposition, we show that the necessary and sufficient condition for $E(e, G) > E(e, F)$, that is, underestimation of the type distribution is profitable for the principal. Otherwise, underestimation decreases the agents' expected efforts and is not profitable for the principal.

**Proposition 8.** *In the situations of underestimation of the type distribution,* $E(e, G) > E(e, F)$ *if and only if* $\int_0^1 (1 - F(\theta))\theta(g(\theta) - f(\theta))d\theta > 0$*.*

*Proof.* This statement follows straightforwardly from Propositions 2 and 5. ∎

We provide two examples. The former shows that underestimation may increase an agent's expected effort and the latter shows that underestimation may decrease an agent's expected effort.

**Example 3.4.** *Suppose that agent i's prior information is* $(a_i, G)$*, where* $a_i = \theta_i$*, and that* $F(\theta) = \theta$*. In the correctly estimated case,* $E(e, F) = 1/6$*.*

- *If* $G(\theta) = (3\theta - \theta^3)/2$*,* $E(e, G) = 7/40$ $(> 1/6)$*.*
- *If* $G(\theta) = 3\theta - 2\theta^{3/2}$*,* $E(e, G) = 11/70$ $(< 1/6)$*.*

## 4 Concluding Remarks

In this paper, we have examined two different sources of overconfidence and have compared the behavioral consequences of each situation with the benchmark case. The main result is that overestimation of one's own ability always induces aggressive behavior by the participants, whereas underestimation of the rival's ability (or the type distribution) sometimes brings about less aggressive behavior by one or both participants. This means that overconfidence does not always induce aggressive behavior. This is quite different from the situations of one person's decision-making.

To conclude the paper, three remarks are in order. First, for an agent, both overestimation of his own type and underestimation of the type distribution increase his subjective probability of winning the contest. For example, consider an agent with true type 1/2 in the situation where $F(\theta) = \theta$. In the correctly estimated case, his probability of winning is 1/2. However, in the situation of overestimation of his own type as $a_i = \sqrt{\theta_i}$, his subjective probability of winning is $\sqrt{1/2}$, which is strictly larger than 1/2. In the situation where he underestimates the type distribution as $G(\theta) = 2\theta - \theta^2$, his subjective probability of winning is 3/4, which is also strictly larger than 1/2. In both situations, he has overestimated the probability of winning the contest. However, these two situations may yield different consequences. The former is always profitable for the principal but the latter may not be.

Second, experiments on economic contests under a situation where participants are overconfident may be interesting. However, they involve some difficulties. In the all-pay auction experiments without overconfidence, Noussair and Silver (2003) observed over-bidding behavior by the subjects.[14] So, if we observe over-bidding (or under-bidding) in experiments with overconfidence, we cannot conclude immediately that overconfidence yields over-bidding (or under-bidding). Consequently, we should design experiments carefully to reach a proper conclusion.

Third, we have shown that different sources of overconfidence have different behavioral consequences. This type of conclusion is not only in situations of contests or auctions. Consider Bertrand competition in a differentiated duopoly. Since strategic complementarities exist, lower marginal costs of one's own firm (i.e., higher own relative ability) yield his aggressive behavior, whereas higher marginal costs of the rival firm (i.e., lower rival's relative ability) yield his less aggressive behavior.

[14] Dorsey and Razzolini (2002) attempted to describe the source of over-bidding behavior of bidders in first-price auctions by auction experiments and compared choices under a first-price auction and an incentive-wise identical lottery. They concluded that the subjects cannot calculate the probability of winning in auctions appropriately, and so over-bidding occurs.

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